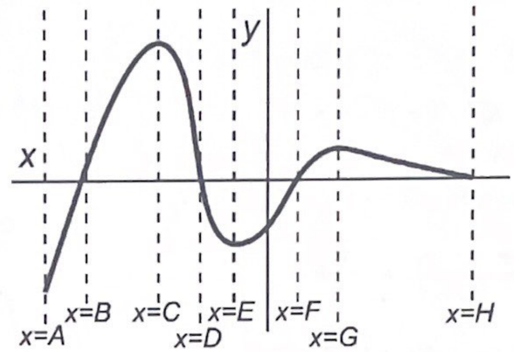
Example 1

To the right is a graph of the function $y = g'(x)$, the *derivative* of the function $g(x)$. The domain of this derivative is the interval [*A*,*H*].



A graph of the function $y = g'(x)$

a. State the intervals in which the function $g(x)$ would be increasing. Give reasoning supporting each of your answers.

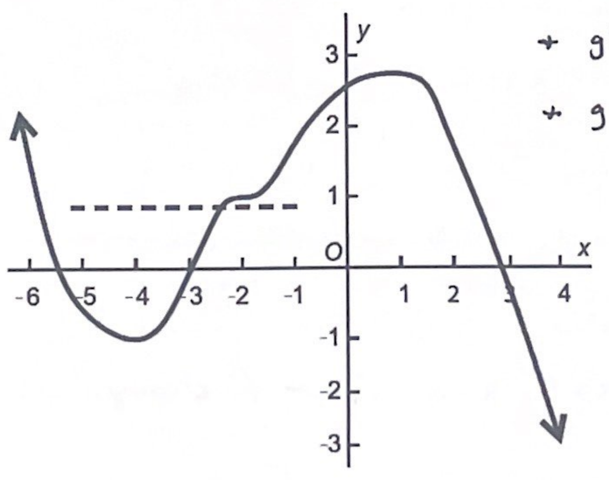
$(B, D), (F, H) \to g' > 0$

b. State the intervals in which the function $g(x)$ would be decreasing. Give reasoning supporting each of your answers.

$(A, B)\ (E, F) \to g' < 0$

Example 2

Suppose that *g* is differentiable. A graph of the *derivative* of *g*, that is, $y = g'(x)$, is displayed below. Use that graph to answer these questions: which *x*-values have $g''(x) = 0$, and what are the first coordinates of any inflection points of $g(x)$?



* $g'' = 0$ @ $x = -4, -2, 1$

* g has P.o.I @ $x = -4, 1$ g'' changes signs.

Graph of $y = g'(x)$, the derivative of $g(x)$)
(This graph has a horizontal tangent at $x = -2$.)

Example 3 (No Calculator)

Let f be the function defined by $f(x)=\frac{1}{3}x^3-4x^2-9x+5$. On which of the following intervals is the graph of f both decreasing and concave down?

$f'(x) = x^2-8x-9=0$

$(x-9)(x+1)=0$

$x=9 \quad x=-1$

$+ \quad - \quad +$

$-1 \quad 9$

Inc: $(-\infty,-1)\ (9,\infty)$

$f''(x) = 2x-8=0$

$x=4$

$- \quad +$

4

Concave down: $(-\infty,4)$

$f(x)$ is dec. & concave down on $(-1,4)$ b/c $f'<0$ & $f''<0$.

Example 4 (Calculator)

The first derivative of the function h is given by $h'(x)=3\ln(2+\cos(2x))-x$, and the second derivative of h is given by $h''(x)=\frac{-6\sin(2x)}{2+2\cos(2x)}-1$. On what open intervals contained in $-2<x<2$ is the graph of h both increasing and concave down?

Inc: $(-2, 1.085)$

C. Down: $(-2,-1.485)(-0.250, 1.655)$

$h(x)$ is increasing & concave down on $(-2,-1.485)$ & $(-0.250, 1.085)$

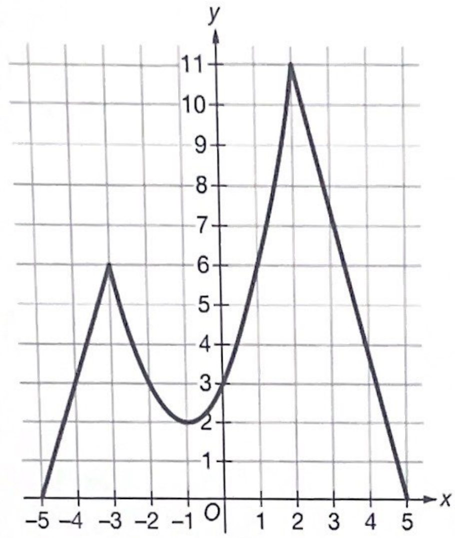
Example 5

x	$0<x<5$	$x=5$	$5<x<8$	$x=8$	$8<x<12$	$x=12$	$12<x<16$
$f'(x)$	Positive	Undefined	Negative	-2	Negative	0	Positive
$f''(x)$	Positive	Undefined	Negative	0	Positive	0	Positive

The function f is continuous on the interval $(0,16)$, and twice differentiable except at $x=5$ where the derivative is undefined. Information about the first and second derivatives of f for values of x in the interval $(0,16)$ is given in the table above. At what values of x in the interval $(0,16)$ does the graph of f have a point of inflection?

f has a P.o.I. @ $x=5$, $x=8$ since f'' changes signs & $f(x)$ is continuous.

Example 6



Graph of f

The continuous function f is defined on the closed interval $[-5,5]$. The graph of f consists of a parabola and two line segments, as shown in the figure above. Let g be a function such that $g'(x)=f(x)$.

(a) Fill in the missing entries in the table below to describe the behavior of $f'(x)$ and $f''(x)$. Indicate "Positive", "Negative", or 0.

	x	$-5<x<-3$	$-3<x<-1$	$-1<x<2$	$2<x<5$
$g'(x)\rightarrow$	$f(x)$	Positive	Positive	Positive	Positive
$g''(x)\rightarrow$	$f'(x)$	Positive	Negative	Positive	Negative
	$f''(x)$	0	Positive	Positive	Zero

(b) There is no value of x in the open interval $(-1,5)$ at which $f'(x)=\dfrac{f(5)-f(-1)}{5-(-1)}$. Explain why this does not violate the Mean Value Theorem.

Since $f(x)$ is not differentiable on $(-1,5)$, MVT does not apply

(c) Find all values of x in the open interval $(-5,5)$ at which the graph of g has a point of inflection. Explain your reasoning.

$g''=f'(x)$

g has a p.o. Inflection at $x=-3$, $x=-1$, $x=2$ b/c $g''(x)$ changes signs.

(d) At what value of x does g attain its absolute maximum on the closed interval $[-5,5]$? Give a reason for your answer.

Since $g'(x)$ is always positive $\rightarrow$ g is always increasing.

$\therefore$ $g(x)$ attains its absolute max @ $x=5$